

Modeling Radiowaves Scattering and Scintillation in Large-Scale Ionosphere Plasma Turbulence Simulations

A Proposal for Jack Eddy Postdoctoral Fellowships

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1 Introduction

Terrestrial weather often has a severe impact on individuals and society. Ground communication, transportation, and public safety, for example, can all be adversely affected. Early warnings of severe weather facilitate early action that can save lives and prevent economic loss. The impact of space weather has now become comparable to that of terrestrial weather, as illustrated by multiple recent incidents, such as the major blackouts in some big cities and GNSS navigation outages during solar storms and sub-storms [1]. The status of Earth's ionosphere reflects the current conditions of the space weather either directly by the precipitation of energetic particles or indirectly by the coupling with the magnetosphere. The plasma turbulence, especially at the low- and high-latitudes, has a severe effect in the accuracy of the navigation systems, which are central to many modern-day applications and activities.

Global Positioning System (GPS) is a part of the Global Navigation Satellite System (GNSS) and is at the core of many applications and technologies nowadays. For example, bank transactions and payment location verification, polar route flights, autonomous vehicles, zero visibility driving, precision agriculture, in addition to other growing applications and technologies that all rely on the accuracy and precision of a GPS receiver in estimating its location. The ionosphere is a major natural source of errors in the GPS navigation system, and knowing the effects of the ionosphere on the transionospheric signals becomes crucial.

2 Ionospheric Irregularities and GNSS Navigation

2.1 Ionosphere Irregularities

The Earth's ionosphere is rich in irregularities in electron density with scale-lengths extending from a few centimeters to hundreds of kilometers at all latitudes [2, 3]. These irregularities are caused by different types of plasma processes and instabilities that are due to the availability of different sources of free energy in the system. Electron precipitation and high-energetic particles at polar regions also cause enhancements in the plasma density and the formation of irregularity structures of sizes ranging from 1 to 75 kilometers [4, 5].

The plasma instabilities are organized into two categories: Macroinstabilities and Microinstabilities, depending on their scale-sizes [8]. The large-scale irregularities have structure of sizes greater than tens of meters to hundreds of kilometers and are generated by macroinstabilities, such as density-gradient [7], temperature-gradient [9], and field-aligned currents [10]. However the microinstabilities are of scale-lengths on the order of or less than the ion Larmor radius [8] and can be excited due to the presence of sharp density-gradient [11] or a velocity-shear [12].

2.2 Transionospheric Delay of GPS Signal

GPS signal propagation through the ionosphere is influenced by the electron density, which causes a variation in the signal group and phase velocities. A corresponding time delay will take place for the signal related to the electron content along the propagation path through the ionosphere, which will give rise to an inaccurate estimate of the location of the receiver [13].

Most of single frequency GPS receivers use the estimated ionospheric delays which is either modeled under a quiet space weather conditions such as Klobuchar model [14], or calculated from real-time maps of electron density such as *International Reference Ionosphere (IRI)* or *Multi Instrument Data Analysis System (MIDAS)* [15]. Although these methods are valid to some extent during the solar quiet times, they are not robust enough in the presence of irregularities of electron density and during solar storms [13].

On the other hand, the undifferentiated dual-frequency GPS receivers calculate the ionospheric delay using the transmitted L_1 and L_2 signals available from the GPS satellites as a part of the Standard Positioning Services (SPS) which is designated for civilian use [16]. Although the inter-frequency biases in both the satellite and receiver are limiting factors in dual-frequency receivers, these biases are stable over a week or more and can be estimated [17]. However, the estimation of the ionospheric delay of the GPS signals using dual-frequency receivers is not reliable in presence of vigorously perturbed plasma regions that interrupt the receiver tracking loops [13, 37].

2.3 Scintillations of GPS Signal

The propagation of a plane-wave through a region of irregularities will perturb the phase of the wavefront depending on the temporal and spatial variations of the refractive index within the irregularities. Also, the power of the GPS signal decreases in some cases to the sky noise level due to the fading of the signal as it propagates through the disturbed plasma regions. In addition, there are multiple constructive and destructive interference processes take place between the layers of irregularities and down to the receiver which give rise to more variations in the signal phase. This is called "*Scintillation*." [13, 18].

Scintillation is most common at high-latitude and low-latitude regions [18]. In the equatorial and low-latitude regions, the amplitude scintillation is strong and coincident with the formation of bubbles in the F-region during the nighttime [19, 21, 22]. However, in the high-latitude region, the enhanced electron irregularities are organized in patches or blobs of high temporal and spatial variations that give rise to strong phase scintillation [20, 23].

A good estimation of carrier-phase and amplitude is at the heart of finding the accurate location of an observer using the GPS constellation. The receiver phase-locked loop (PPL) helps in estimating the location by counting the number of cycles between the receiver and the space vehicles that transmit the navigation signals. The scintillating signals of deep power fades and turbulent phase variations might cause cycle slipping or loss-of-lock with the satellite. Any interruption in the tracking loops gives rise to an inaccurate estimation of the receiver location until another lock takes place.

In [41], the authors attributed the failure of the phase-locked loop to track the scintillating signals to three main reasons: (1) an increase in the phase error variance, (2) a cycle slipping, and (3) a frequency unlock. A fixed-interval smoothing technique has been used by the authors in [37] for a non-real-time receiver to track scintillating signals and no loss-of-tracking in the phase-locked loop took place. However, this method can not be used in real-time receiver because it requires a post-processing of all recorded observations in an interval of time to estimate the state of the system during strong scintillations.

The design of a real-time receiver that is robust during strong scintillations needs an integration between the three components in the navigation system: (1) A physics-based plasma turbulence model to retrieve the characteristics of the ionosphere irregularities, (2) a physics-based propagation model to quantify the level of scintillation in the propagating signals, and (3) a GPS receiver-simulator to evaluate the robustness of its carrier tracking loop under different solar and geophysical conditions.

3 Available Solutions and their Limitations

There are a number of physics-based and statistics-based models used in estimating the scintillations and testing the robustness of the GPS receiver under weak and strong scintillation conditions.

3.1 Statistics-Based Solutions

The robustness of the GPS receiver carrier tracking loop against scintillation can be tested directly by employing a statistical model. A time-history of the scintillation data have been generated empirically or synthetically with different statistical approaches for their limitations to be tested in many literatures [36, 39, 40]. A more realistic data-driven simulation testbed based on a library of empirical phase and amplitude time histories recorded under a wide range of equatorial scintillations [40, 41]. Although the statistical models reflect the effect of the amplitude and phase scintillations on the carrier tracking loops, they do not show the connection between that effect and its physics-based cause which questions its capabilities for forecasting.

3.2 Physics-Based Solutions

The authors in [24, 26] reviewed several physics-based propagation models for the transionospheric radio channel that has been developed in order to quantifying the amplitude and phase scintillations caused initially by the ionospheric irregularities.

Ionospheric climatology models, such as WBMOD in Wideband Ionospheric Scintillation Model [27], NeQuick in Global Ionospheric Scintillation Propagation Model (GISM) [23, 25], and NeQuick and EP-PIM in the Hybrid Scintillation Propagation Model [28–32], are employed for the variations of the plasma density with the seasons, solar activity, and space weather conditions to be considered. The ionosphere irregularities of different scales are modeled empirically in the simulations based on their characteristics from the observations at the low-latitude [32] and high-latitude [31] regions.

The authors in [33, 34] classified the signal scintillations into three categories: weak, medium, and strong. While the complex-phase-screen and random-field-screen techniques are used to quantify the weak and strong scintillations, respectively, in the hybrid propagation model [28–32], a multiple-phase-screens technique is used in the global ionospheric propagation scintillation model [24].

Although the good agreement between the scintillation maps simulated in the propagation models mentioned above and the measurements from different network of GPS receivers, the link between the physics-based models of the ionosphere plasma irregularities that cause the scintillations and the effect of these scintillations in the tracking loops of the GPS receivers is still missing.

4 A Proposed Fully Physics-Based Solution

The project proposed for this Jack Eddy Postdoctoral Fellowship application seeks to close the gap between the physics of the ionosphere plasma irregularities and the engineering of the signal tracking in the global positioning system receiver. This could happen by the integration of 1) a physics-based model for the plasma irregularities of effective scales that cause both amplitude and phase scintillations in the GPS signals, 2) a physics-based model for the signal propagation from the satellite to the receiver through turbulent plasmas in the ionosphere, and 3) a physics-based simulator for the GPS receiver that catches and processes the modeled propagated signal to check the cases that cause failures in the receiver tracking loops and work in modifying and fixing it.

To achieve the preliminary goals of a project that integrates these three aspects in two years, the research plan divides this project into three modules to be developed concurrently but working dependently, and to be summarized below:

4.1 Physics-Based Model for the Effective Multi-Scale Ionosphere Irregularities

As a part of my graduate research, we have developed a 3D solver for a system of first-order coupled partial differential equations to study the instabilities in the equatorial electrojet. The solver uses both Pseudospectral (with periodic boundary conditions) and Chebyshev (with non-periodic boundary conditions) methods to find the spatial derivatives of the evolving fields in the dynamic equations with homogeneous or heterogeneous boundary conditions. For the time integration of the dynamic system I use a fifth- and sixth-order Runge-Kutta method with adaptive time-step. The solver is initialized by solving the eigenvalue problem and initialize the fields in the system in the spectral domain with the eigenvectors.

For the ionosphere background parameters, the code employs either the empirical models such as *International Reference Ionosphere (IRI)*, *Mass Spectrometer and Incoherent Scatter Radar Exosphere (MSISE)*, and *Horizontal Wind Model (HWM)*, or self-consistent global ionosphere models such as *Thermosphere Ionosphere Electrodynamics General Circulation Model (TIEGCM)*.

Currently, the code is being upgraded to include *Message Passing Interface (MPI)* parallelization in order to facilitate faster operation and larger problem sizes (the current version utilizes multiple-cores using OpenMP). This will allow us to use the code in resolving multi-scale instabilities in a *three-dimensional* simulation box that covers a large portion of the ionosphere.

The plasma instabilities in the F-region have been observed and studied extensively for more than six decades []. As mentioned above, it was found that the F-region during the nighttime is a major source of the amplitude and phase scintillations at the low-latitude region []. Immediately after the start of the postdoctoral fellowship, a 3D fluid model for the instabilities in the bottomside and topside of the equatorial and low-latitude F-region will be studied and modeled. Thereafter, the dynamic equations of the model will be evolved in numerical simulations under different space weather and geophysical conditions in order to quantify the temporal and spatial variations of the electric fields and electron density structured in the plasma irregularities.

The numerical results will be evaluated and validated with both the observed measurements and other numerical simulation results. In addition, the physical model of the F-region instabilities will be revisited frequently to include other factors that may influence the instabilities and to check their effects on the signal scintillations.

4.2 Physics-Based Model for the Signal Propagation through Irregularities

This module of the project is to be divided into two submodules: (1) ray-tracing and homing-in of transionospheric signal, and (2) signal amplitude and phase scintillations in multiple phase-screens (MPS).

4.2.1 Haselgrove Ray-Tracing Equations

In a series of papers [43–45] Haselgrove used a Hamilton [42] representation of the optical rays as a solution of a variational problem to discuss the derivation and applicability of the Hamiltonian equations of geometrical optics as a transionosphere ray tracing technique.

Numerical solution of the Haselgrove equations provides a technique for calculating, in addition to the path of the principal ray, the Doppler shifts, signal power, phase-path, group-path, and angle-of-arrival. The calculation of these quantities allows us to estimate the ionospheric delay of the GPS signal due to the modeled and numerically simulated ionosphere irregularities. In addition, the nature of the Hamiltonian system allows one to use ray-tubes from the transmitter to home-in the receiver [46, 47]. Moreover, there are a broad range of applications of the Haselgrove equations in the ionosphere and magnetosphere at different operating frequencies [47–49].

4.2.2 Multiple Phase Screens

After solving Haselgrove equations for the ray path from the satellite to the receiver and calculating the error parameters, the statistical facts of the results from the numerical simulations of the ionosphere plasma irregularities will be used as a guide or baseline to form the multiple phase screens (MPS) required for calculating the amplitude and phase scintillations.

Based on Ratcliffe [50], Knepp [51] proposed a multiple phase screen model to resolve the parabolic equation to quantify the signal distortion by the time-selective and frequency-selective properties of the disturbed ionosphere. This method models the effect of the random variations of the refractive index within the plasma irregularities and the angular scattering which causes constructive and destructive interference of the spectral contents of the propagating signal that give rise to amplitude and phase scintillations. This technique is discussed and employed in the GISM model by Beniguel [23] but they use it for multiple phase screens generated statistically based on empirically models of observations.

Employing the multiple phase screens in our propagation model will help in initializing them based on numerically simulated data either from different models for ionosphere irregularities or several simulations of the same model but at different altitudes and background conditions. For example, it will be possible to include multi-scale irregularities from the bottom of the E-region to the topside of the F-region and even as high as the plasmasphere for different types of instabilities and plasma processes to be resolved.

The estimated amplitude and phase scintillations from the propagation model are more realistic as they rely on physics-based models for the plasma instabilities. This gives us a new dimension in studying the scintillation of the radiowaves signals due to the ionosphere irregularities of different effective scales.

4.3 GPS Receiver Simulator

The GPS software receivers have been discussed, implemented, and tested as reported by many authors in [41,52–54]. The design of software receivers is flexible for modifications which are good for conducting research and doing experiments. Also, it is easy to test the status of the signal at many checkpoints as it is processed in the software receivers.

The scintillating signal-like output from the propagation model will be manipulated to have it in the form that is suitable to be used as an input for the GPS software receiver simulator with no need for its front-end. This allows us to test the performance of the receiver components under different levels of scintillations for many space weather and geophysical conditions. In addition, the robustness of different designs of the software receiver tracking loops under weak, medium, and strong levels of scintillations is going to be tested to achieve the optimized design and its fields of applicability.

5 Summary

We will use Python to develop the code of the propagation model and GPS simulator because it is portable, readable, and easy to use with data from different sources of simulation models in the space weather community.

The integration between the three modeled components of the navigation system; the irregularity, signal scintillation, and GPS receiver, allows better understanding of the system. The high cost in time of the simulation of multi-scale ionosphere irregularities that is driven by satellite data of the current space weather conditions is the limiting factor for real-time forecasting. However, the ability to run many physics-based simulations for multiple regions of the ionosphere and at different conditions of seasons, solar activities, etc. and integrate all of them in one code will give us a large library of simulated data that can be used as a tool for forecasting in similar future cases. With continuing advances in computational resources, this integrated software will become a ready-to-use tool that is capable of real-time forecasting of the space weather conditions in the near future.

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